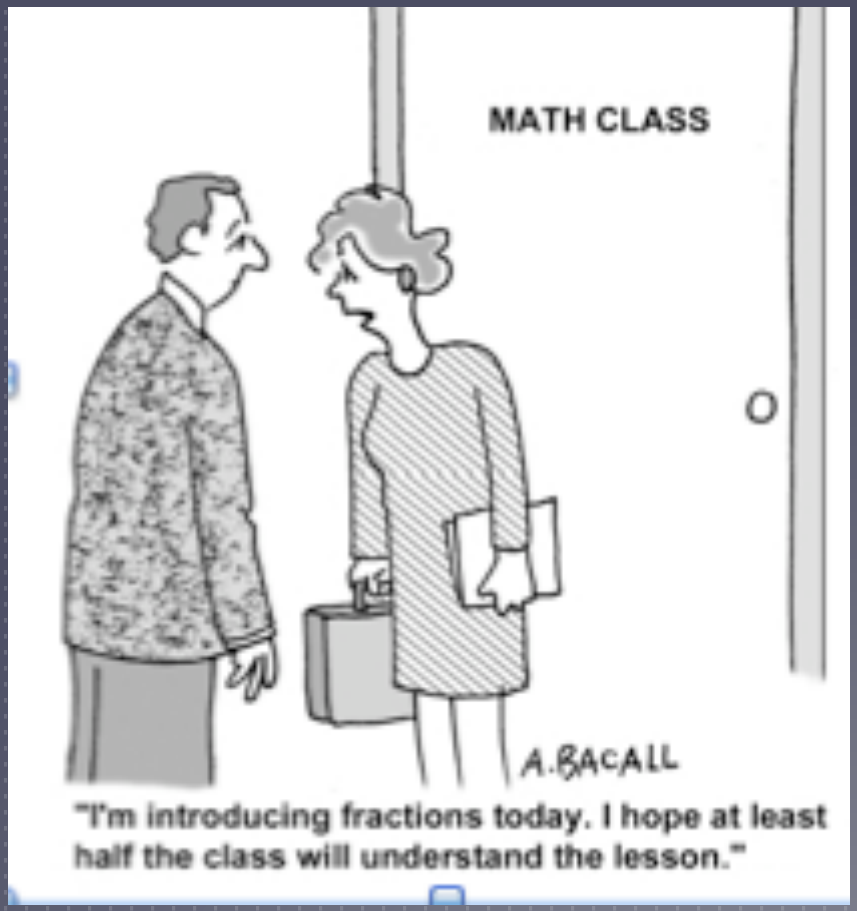
1/2 –WAY THERE:

Getting the Whole Picture of Fractions

RTI Conference 2015

Anchorage, AK

9:00 – 11:30am


MATH CLASS
A.BACALL
"I'm introducing fractions today. I hope at least half the class will understand the lesson."

"Don't worry about Tommy. 5 out of 3 kids his age can't do fractions."

WHAT DO YOU KNOW ABOUT FRACTIONS?

Take a moment to list all of the things that you know about fractions.

- 1 min to think
- 2 mins to write and talk to a shoulder partner
- 3 mins to discuss at the table

WHAT DO STUDENTS KNOW ABOUT FRACTIONS?

- " I know it is one number sitting on top of the other."
- "One number lives upstairs and one number lives downstairs."
- "I know they're less than 1."
- "I know the number underneath has to be the biggest."
- "What I know is that they are really hard."
- "What happens upstairs has to happen downstairs."
- "Alls I know is that if the number on top is bigger than the number on the bottom, the number is *indecent!*"

Penny Williams, ASD, 2015

WHAT DO I KNOW ABOUT FRACTIONS?

Number of those pieces

$\frac{2}{3}$

What size piece are we working with?

How many equal pieces?

BASIC UNDERSTANDINGS FOR FRACTIONS

- Sharing means equal parts
- To compare fractions, you need the same "whole"
- Fewer pieces of the same whole means each piece is larger; more pieces mean smaller pieces
- To compare fractions, you have to look at the numerators AND the denominators
- Equivalent fractions have equal values and proportional numerators and denominators
- Fractions are numbers

FRACTIONS IN THE NEW STANDARDS

3rd Grade

- 3.NF: Develop understanding of fractions as numbers.

4th Grade

- 4.NF: Extend understanding of fraction equivalence and ordering.
- 4.NF: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

5th Grade

- 5.NF: Use equivalent fractions as a strategy to add and subtract fractions.
- 5.NF: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Alaska English/Language Arts and Mathematics Standards, June 2012

2nd Grade

2.G.3. Partition circles and rectangles into shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

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3rd Grade	4th Grade	5th Grade
▸ 3.NF: Develop understanding of fractions as numbers.	▸ 4.NF: Extend understanding of fraction equivalence and ordering. ▸ 4.NF: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.	▸ 5.NF: Use equivalent fractions as a strategy to add and subtract fractions. ▸ 5.NF: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

6th Grade

6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions (e.g., by using visual fraction models and equations to represent the problem).

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Concept Harvest

Individually:

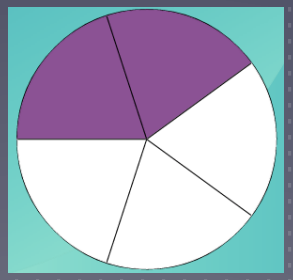
- Represent ¾ in as many ways as you can in 1 minute
- Put each representation on a different post-it note.

As a table group:

- Sort the post-its from the table & place on chart paper

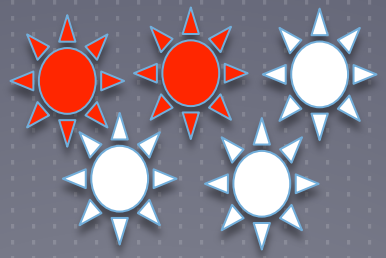
3 Ways to Represent Fractions

- Area Model

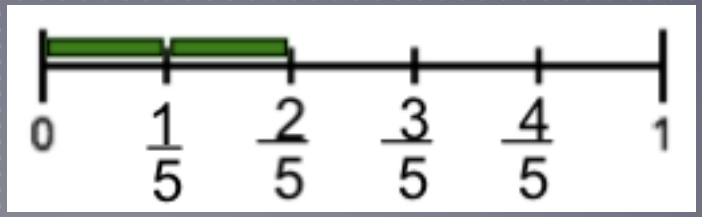


$\frac{2}{5}$

- Set Model



- Linear Model



2 BIG SHIFTS W/ THE NEW STANDARDS

- **Representing fractions on a number line**

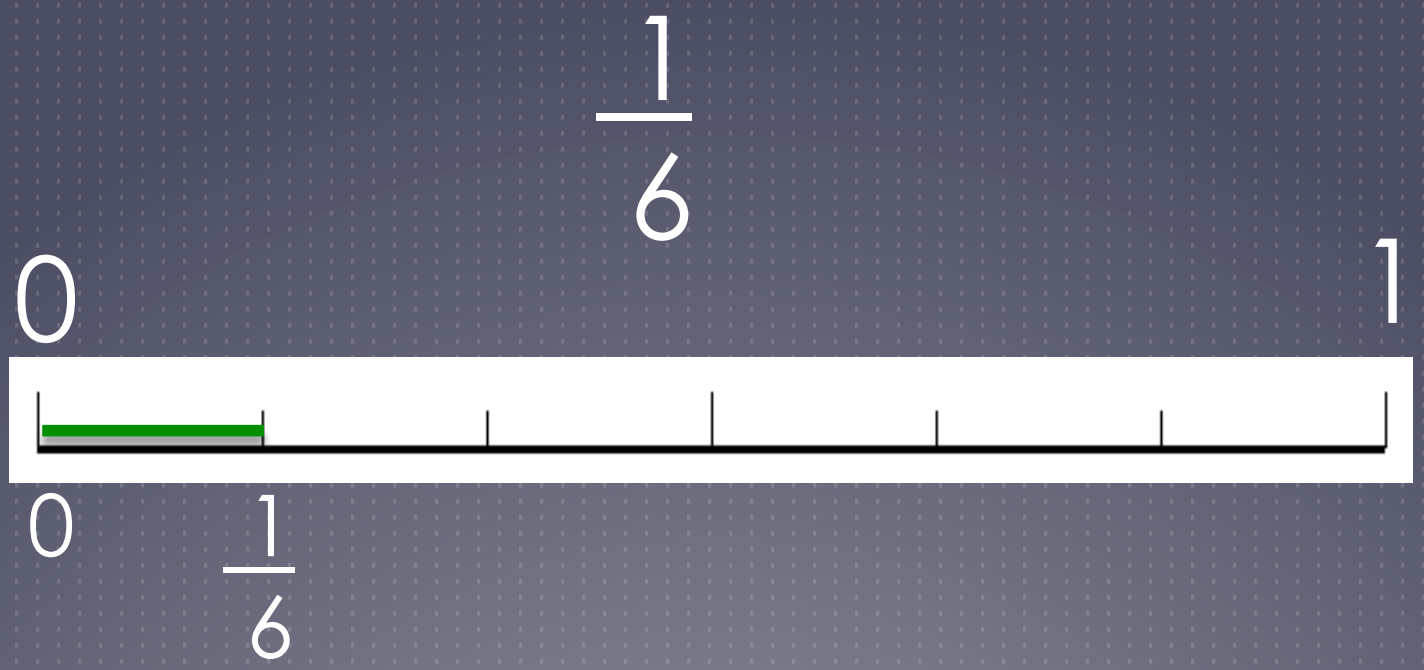
- Seeing unit fractions as the building blocks of all fractions

3.NF: Develop understanding of fractions as numbers

3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction 1/*b* (e.g., 1/6) on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* (e.g., 6) equal parts. Recognize that each part has size 1/*b* (e.g., 1/6) and that the endpoint of the part based at 0 locates the number 1/*b* (e.g., 1/6) on the number line.

UNIT FRACTIONS AS BUILDING BLOCKS



3.NF: Develop understanding of fractions as numbers

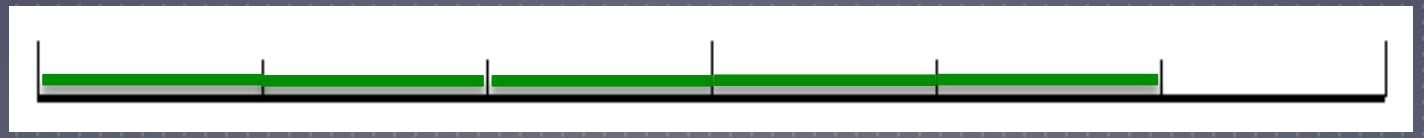
3.NF.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

b. Represent a fraction *a*/*b* (e.g., 2/8) on a number line diagram or ruler by marking off *a* lengths 1/*b* (e.g., 1/8) from 0. Recognize that the resulting interval has size *a*/*b* (e.g., 2/8) and that its endpoint locates the number *a*/*b* (e.g., 2/8) on the number line.

UNIT FRACTIONS AS BUILDING BLOCKS

$\frac{5}{6}$

0 1



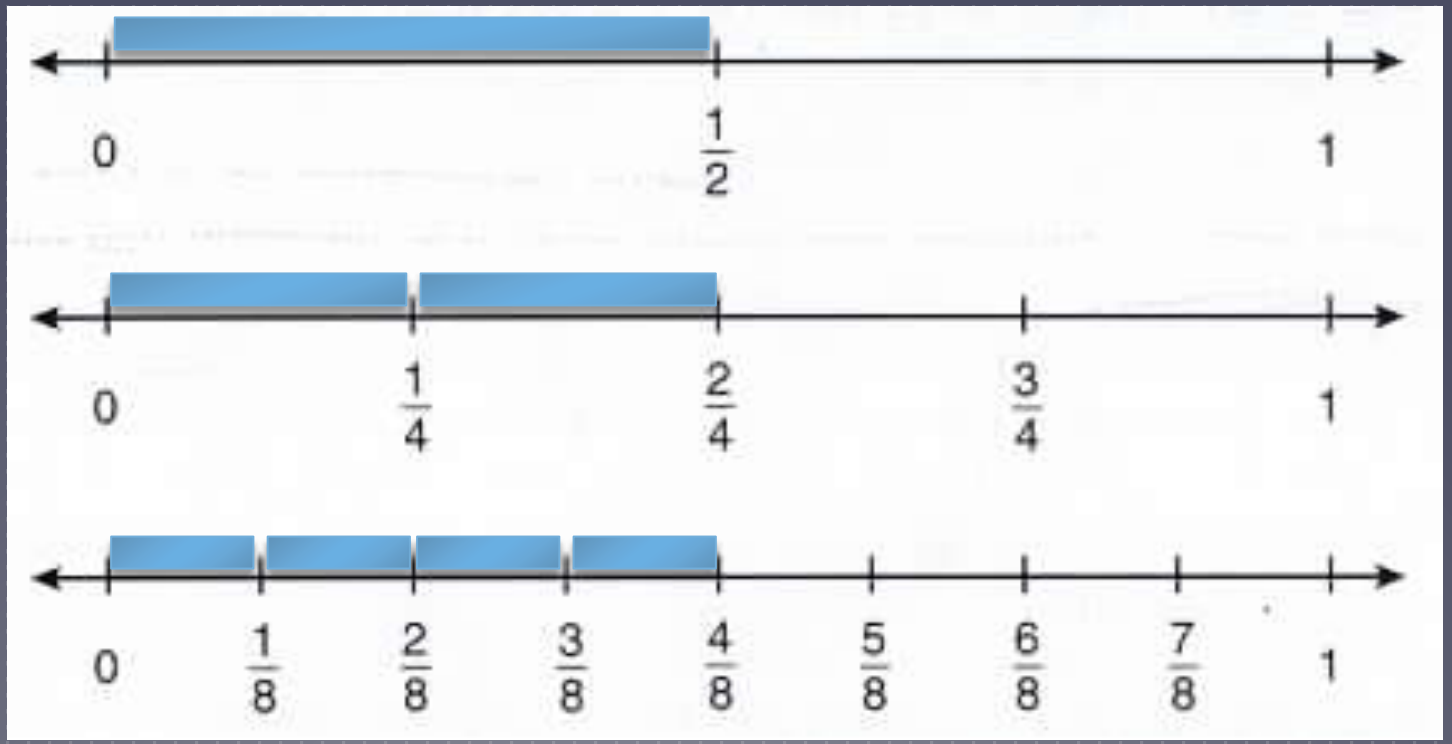
0 $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$

Equivalent Fractions w/Linear Model

Halves (1/2s)

Fourths (1/4s)

Eighths(1/8s)

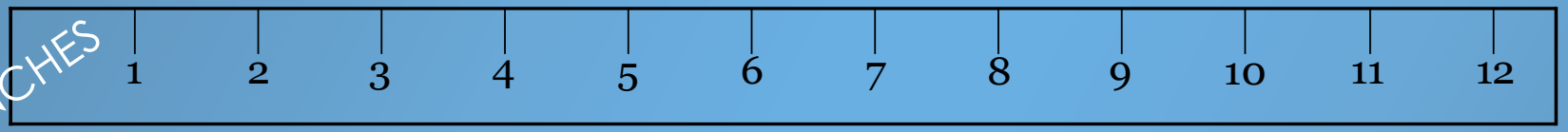


Connections to Measurement

GLE [6[MEA 1 & 3&7
GLE [5[MEA 1 & 8
GLE [4[MEA 1 & 5
GLE [3[MEA 1 & 6

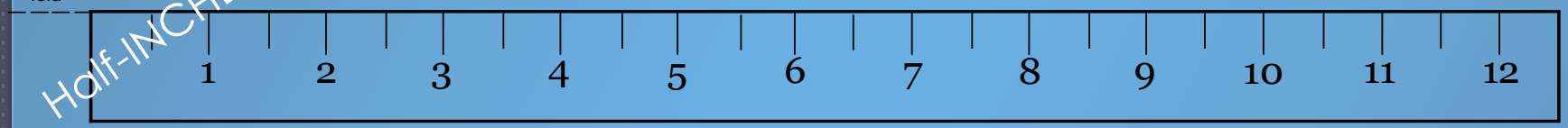
Run on legal paper and fold on lines or run on a transparency (will not go full 12 inches) for each student.

fold



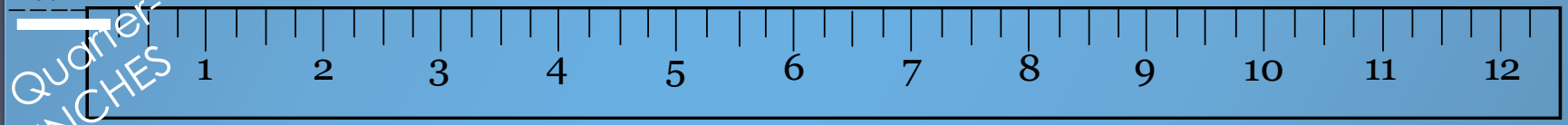
One inch segments

fold



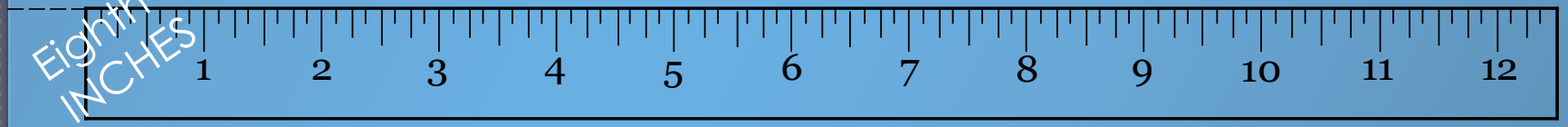
Half inch segments

fold



Quarter inch segments

fold



Eighth inch segments

3.NF: Develop understanding of fractions as numbers – Potential Roadblock

Movie: "Placing ½ on the Number Line"

https://mathsolutions.wistia.com/projects/r4bjpdzb31

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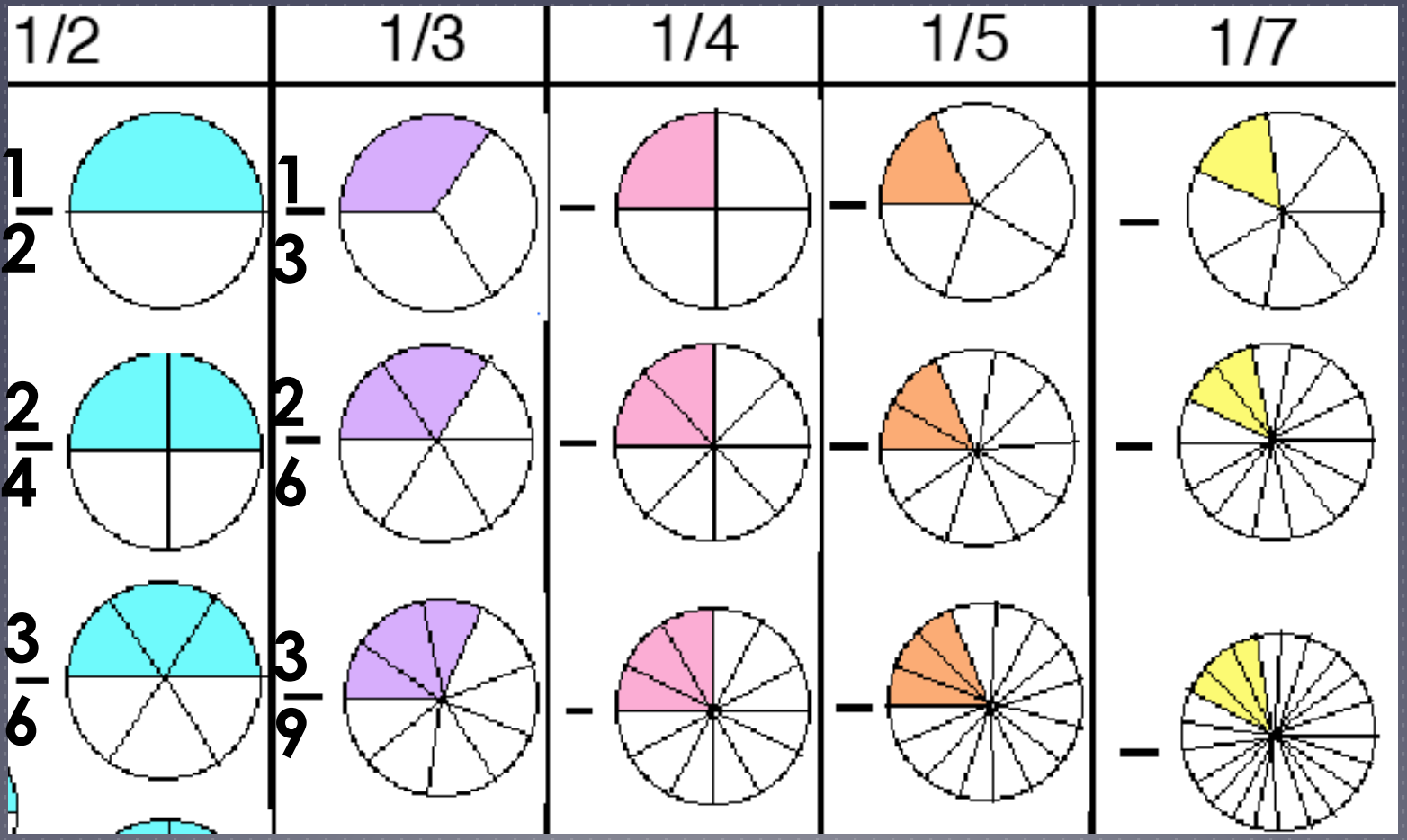
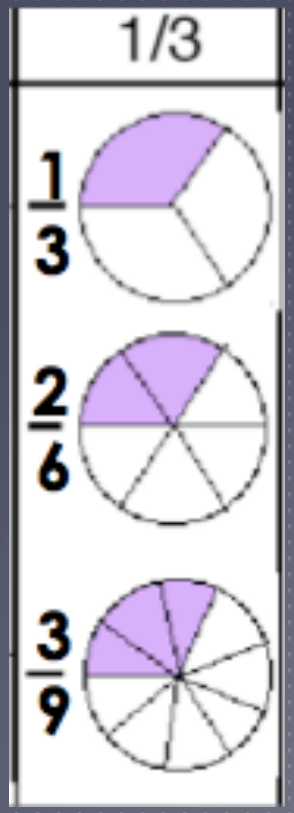
Equivalent Fractions w/Area Model – 3rd

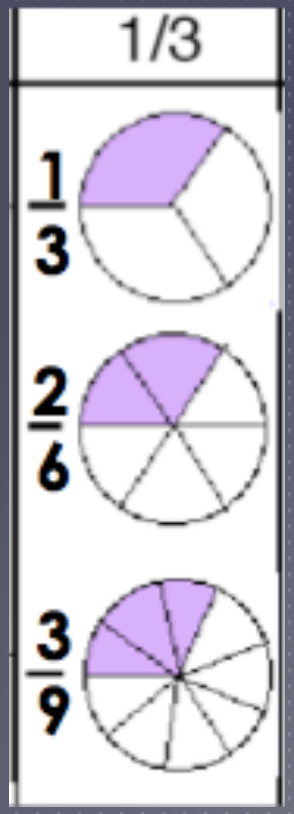


4.NF: Extend understanding of fraction equivalence and ordering.



4.NF.1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF: Extend understanding of fraction equivalence and ordering.



How many are shaded?

Divided into how many pieces?

$\frac{1}{3}$

2 times as many are shaded.

Divided into 2 times as many pieces

$\frac{2}{6} = \frac{1}{3} \times \frac{2}{2}$

3 times as many are shaded.

Divided into 3 times as many pieces

$\frac{3}{9} = \frac{1}{3} \times 3$

Algebra Concepts: Rates & Ratios

The measures of 2 supplementary angles are in a ratio of 1:3....

$$\frac{1}{3} \times \frac{n}{n} = \frac{1n}{3n}$$

$$1n + 3n = 180$$

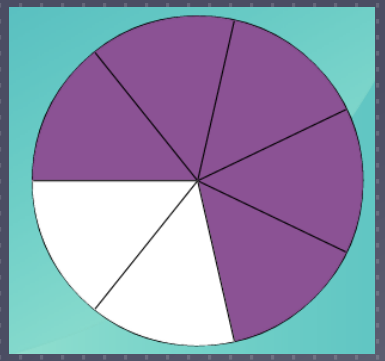
THINK, PAIR, SHARE

Arrange the following numbers in order from least to greatest:

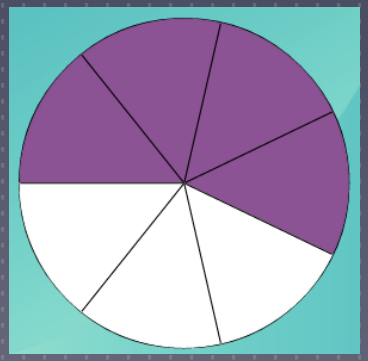
$$\frac{1}{2} \qquad \frac{2}{7} \qquad \frac{5}{9}$$

Explain your reasoning.

Comparing Fractions – 3rd Grade

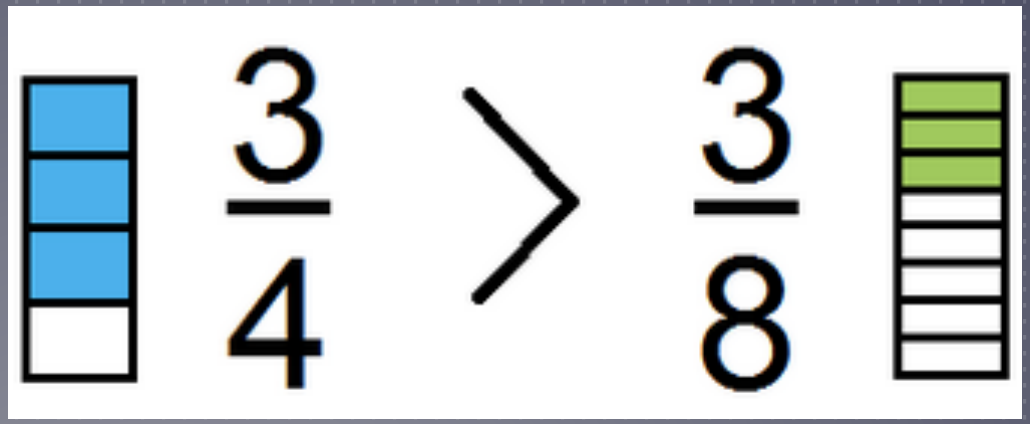


$\frac{5}{7} > \frac{4}{7}$



Same denominators

Same numerators

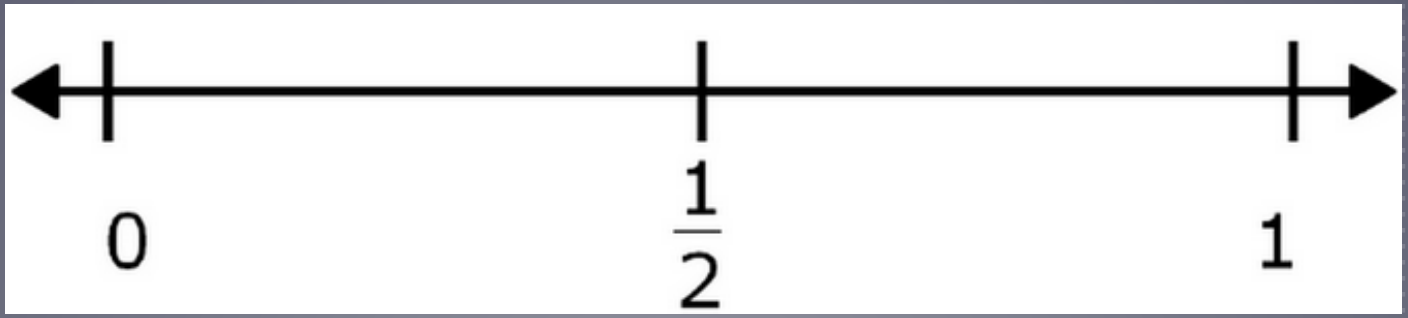


4.NF: Extend understanding of fraction equivalence and ordering.

4.NF.2. Compare two fractions with different numerators and different denominators (e.g., by creating common denominators or numerators, **or by comparing to a benchmark fraction**

Using Benchmark Fractions

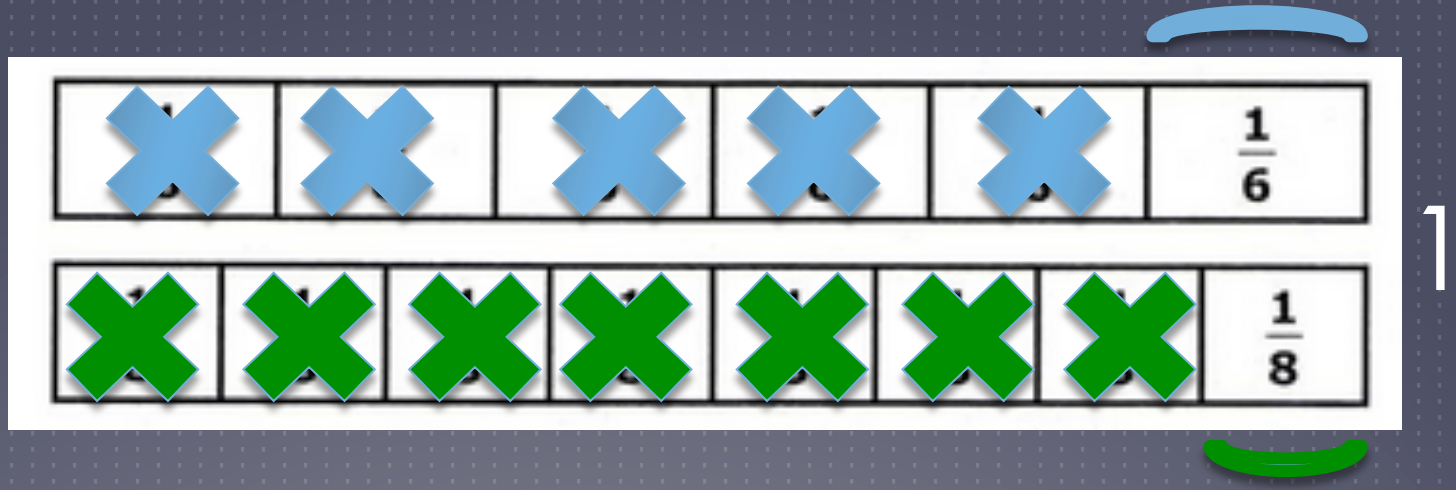
$$\frac{6}{10} > \frac{3}{8}$$



Using Benchmark Fractions

$\frac{5}{6}$

$\frac{7}{8}$



$\frac{7}{8} > \frac{5}{6}$

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2 BIG SHIFTS W/ THE NEW STANDARDS

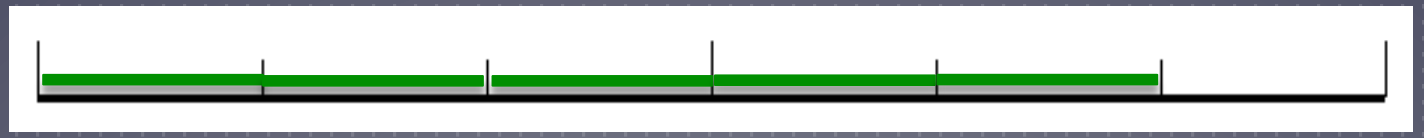
- Representing fractions on a number line

- **Seeing unit fractions as the building blocks of all fractions**

UNIT FRACTIONS AS BUILDING BLOCKS

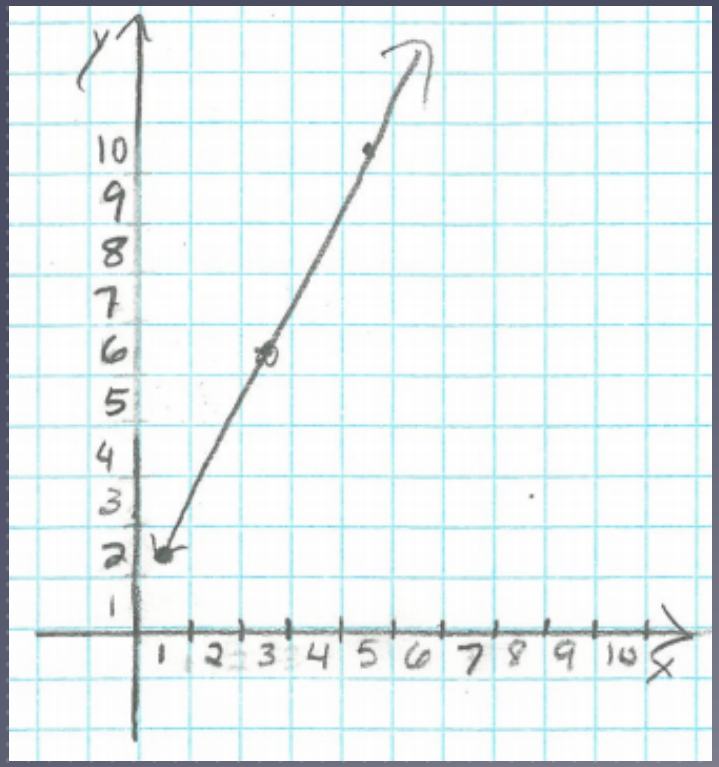
$\frac{5}{6}$

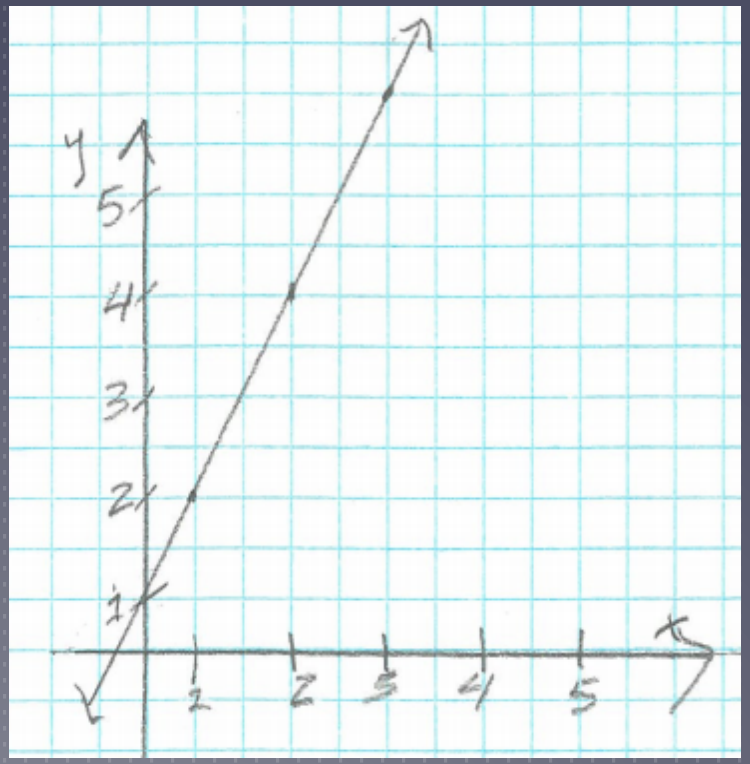
0 1



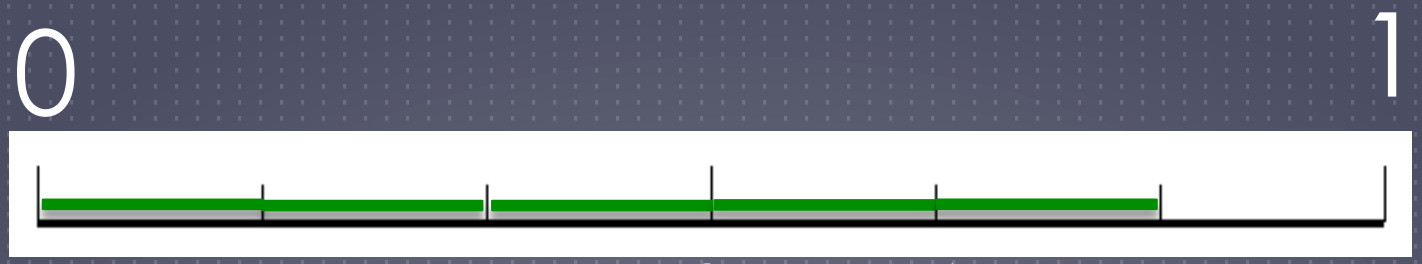
0 $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$

Algebra Concepts: Graphing





4.NF.3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.



$$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 5 \times \frac{1}{6}$$

4.NF.3.a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

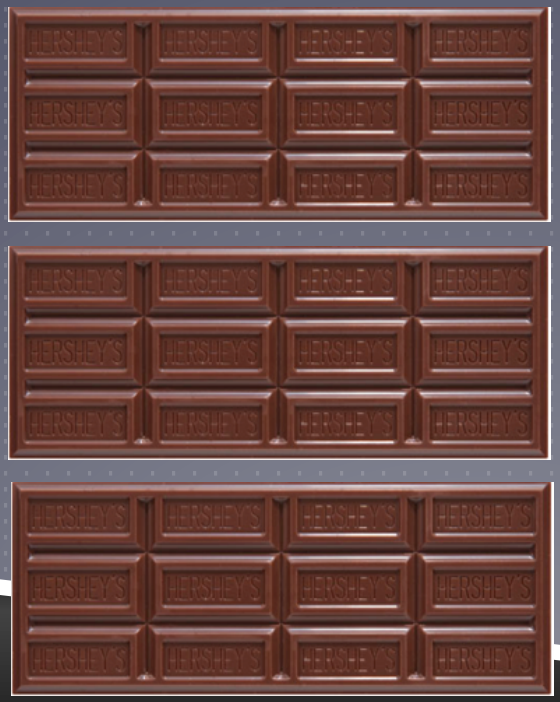
$$\frac{5}{6} = \frac{1}{6} + \frac{4}{6} \qquad \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$$

4.NF.3.b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.

$$\frac{5}{6} = \frac{1}{6} + \frac{4}{6} \qquad \frac{5}{6} = \frac{2}{6} + \frac{3}{6}$$

MIXED NUMBERS & IMPROPER FRACTIONS

$$3\frac{1}{4} = 3 + \frac{1}{4}$$



MIXED NUMBERS & IMPROPER FRACTIONS

$$3\frac{1}{4} = \frac{12}{4} + \frac{1}{4} = \frac{13}{4}$$

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5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

CONNECT: CONCRETE ←→ SYMBOLIC

$\frac{3}{4} \times \frac{3}{3}$ + $\frac{2}{3} \times \frac{4}{4}$

$\frac{9}{12}$ + $\frac{8}{12}$ = $\frac{17}{12}$

$1\frac{5}{12}$

THINK, PAIR, SHARE

Write down 3 different pairs of fractions that will add to a result **close** to ¾ . DO NOT actually do the addition!

Explain your reasoning.

5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and check the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2*

WORD PROBLEMS WITH FRACTIONS

Ludmilla and Lazarus each have a lemon. They need a cup of lemon juice to make hummus for a party. Ludmilla squeezes ½ cup from hers and Lazarus squeezes ⅖ of a cup from his. How much lemon juice do they have? Is it enough?

$$\frac{1}{2} + \frac{2}{5}$$

$$\frac{1}{2} + \frac{2}{5}$$

Estimate.

½ + ≈ ½

≈ 1

Since ⅖ < ½, then I know it's just less than 1.

Calculate.

$$\frac{1}{2} + \frac{2}{5} = \frac{9}{10}$$

$$\frac{1}{2} + \frac{2}{5} = \frac{3}{7}$$

Check Reasonableness.

$\frac{9}{10}$ Is just less than 10/10, so it is just less than 1.

$\frac{3}{7}$ **Is way less than 7/7, so it is way less than 1. In fact, it is less than ½.**

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4th Grade: Connections to Multiplication

$$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 5 \times \frac{1}{6}$$

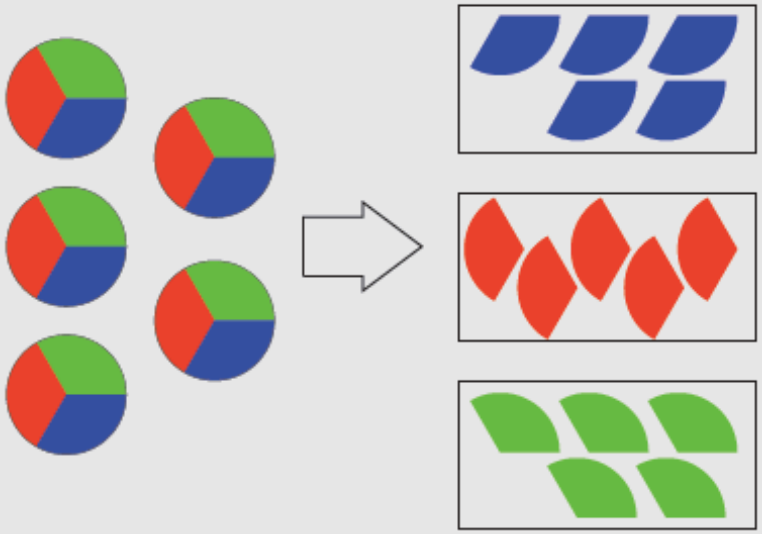
5th Grade: Connections to Multiplication

$$\frac{1}{6} \times 5$$

5.NF.3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$).

How to share 5 objects equally among 3 shares:

$$5 \div 3 = 5 \times \frac{1}{3} = \frac{5}{3}$$



If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute $\frac{1}{3}$ of itself to each share. Thus each share consists of 5 pieces, each of which is $\frac{1}{3}$ of an object, and so each share is $5 \times \frac{1}{3} = \frac{5}{3}$ of an object.

5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts;

$\frac{1}{4} \times 5$ Is 1 part when 5 is partitioned into 4 equal parts.

$\frac{1}{4}$ of 5

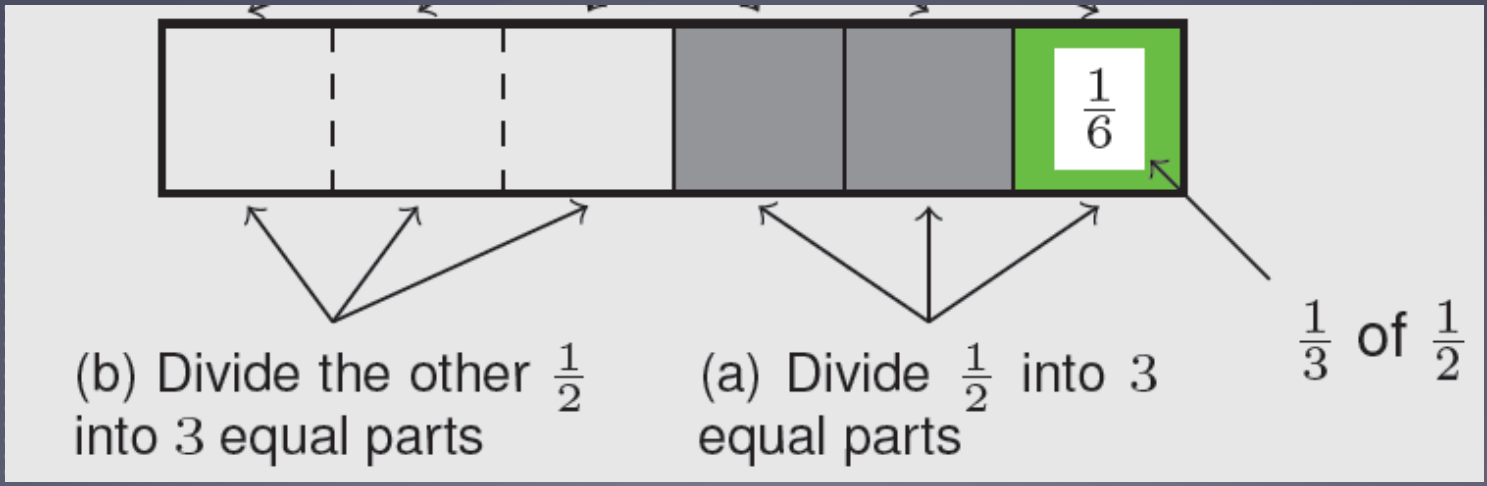
$\frac{1}{4} \times 5 = \frac{1}{4}$ of 5

Share to 4 equal groups.

$$\frac{1}{4} \times 5 = \frac{1}{4} \text{ of } 5$$

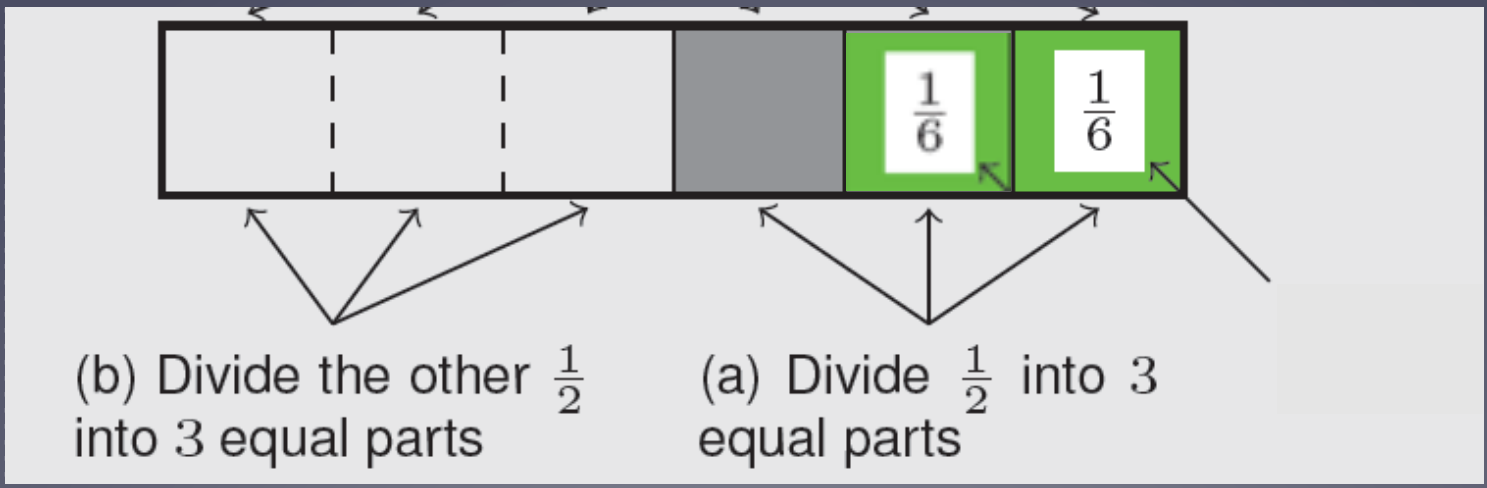
Divide each whole into 4 equal parts. Take 1 part from each.

$\frac{1}{3}$ of $\frac{1}{2}$ Is 1 part when ½ is partitioned into 3 equal parts.



$$\frac{1}{3} \times \frac{1}{2} = \frac{1 \times 1}{3 \times 2}$$

$\frac{2}{3} \times \frac{1}{2}$ Is **2** parts when ½ is partitioned into 3 equal parts.



$$\frac{2}{3} \times \frac{1}{2} = \frac{2 \times 1}{3 \times 2}$$

THINK, PAIR, SHARE

Use a number line to show that:

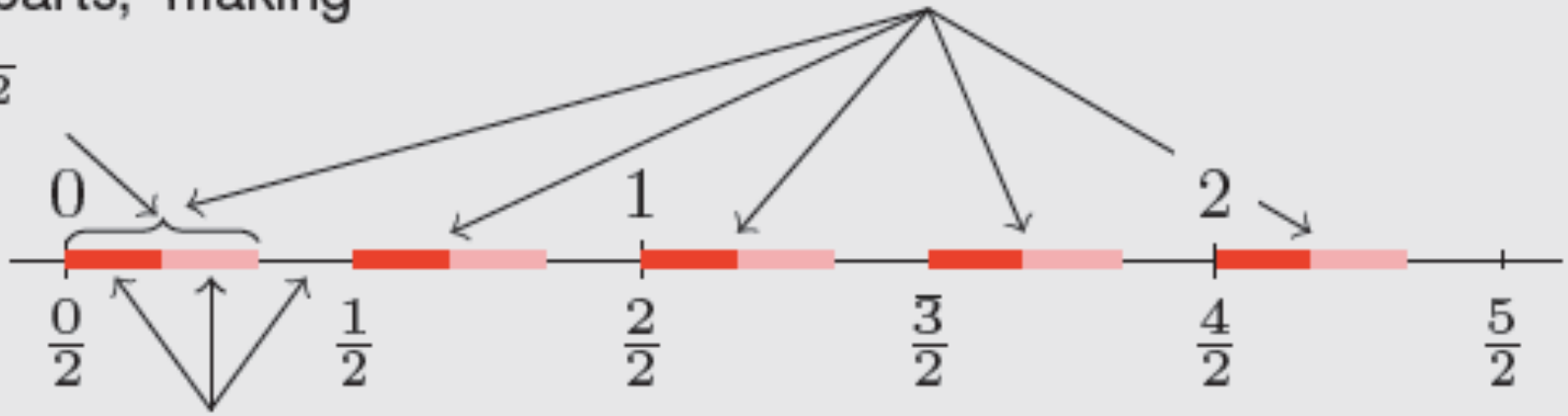
$$\frac{2}{3} \times \frac{5}{2} = \frac{2 \times 5}{3 \times 2}$$

Using a number line to show that $\frac{2}{3} \times \frac{5}{2} = \frac{2 \times 5}{3 \times 2}$

2/3 of 5/2

(c) There are 5 of the $\frac{1}{2}$s, so the segments together make $5 \times (2 \times \frac{1}{3 \times 2}) = \frac{2 \times 5}{3 \times 2}$

(b) Form a segment from 2 parts, making $2 \times \frac{1}{3 \times 2}$



(a) Divide each $\frac{1}{2}$ into 3 equal parts, so each part is $\frac{1}{3} \times \frac{1}{2} = \frac{1}{3 \times 2}$

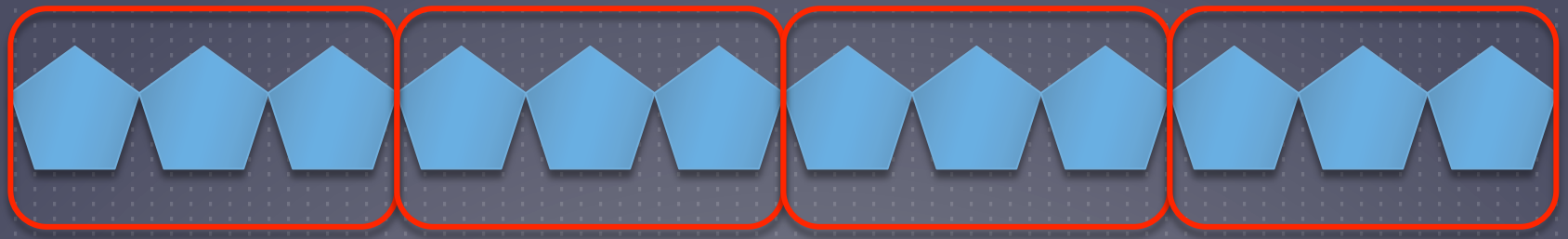
Division! --

5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

- division of a unit fraction by a non-zero whole number
- division of a whole number by a unit fraction

12 ÷ 3 =

How many sets of 3 can be partitioned out of 12?

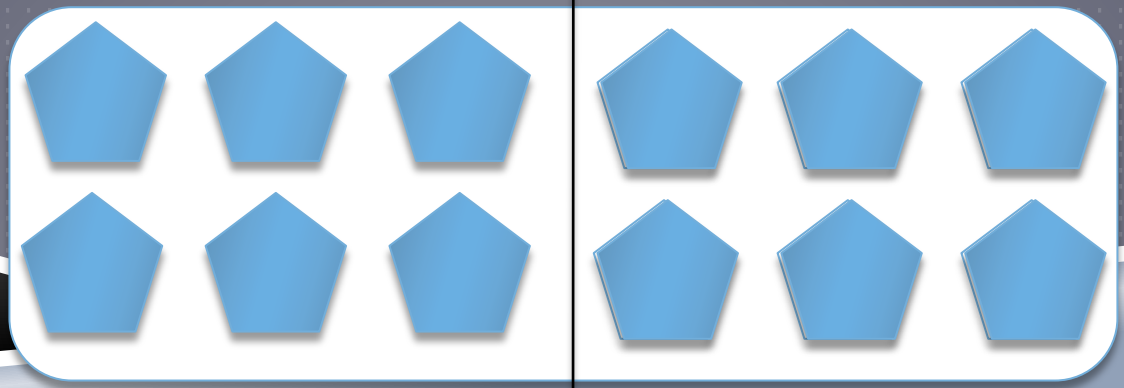


If you share 12 out into 3 sets, how many are in one whole set?

6 ÷ 1/2 =

How many sets of ½ can be partitioned out of 6?

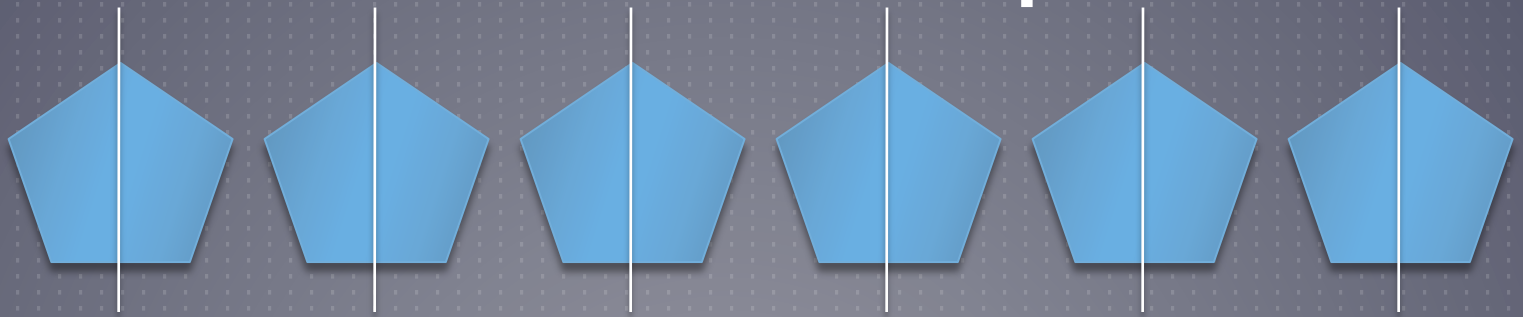
If you share 6 out into 1/2 of a set, how many are in one whole set?



Why flip and multiply?

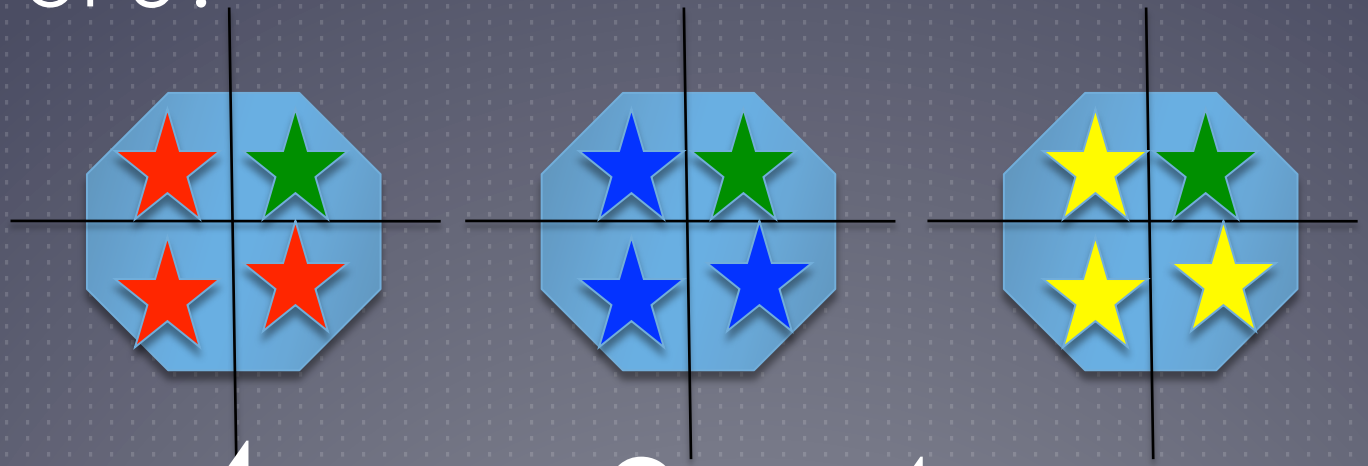
$$6 \div 1/2 =$$

$$6 \times \frac{2}{1} = \frac{6 \times 2}{1}$$



$3 \div 3/4 =$

How many ¾ s can we partition out of 3?



$$3 \times \frac{4}{3} = \frac{3 \times 4}{3} = \frac{12}{3}$$

Can you estimate the answer?

$17 \div 3/4 =$

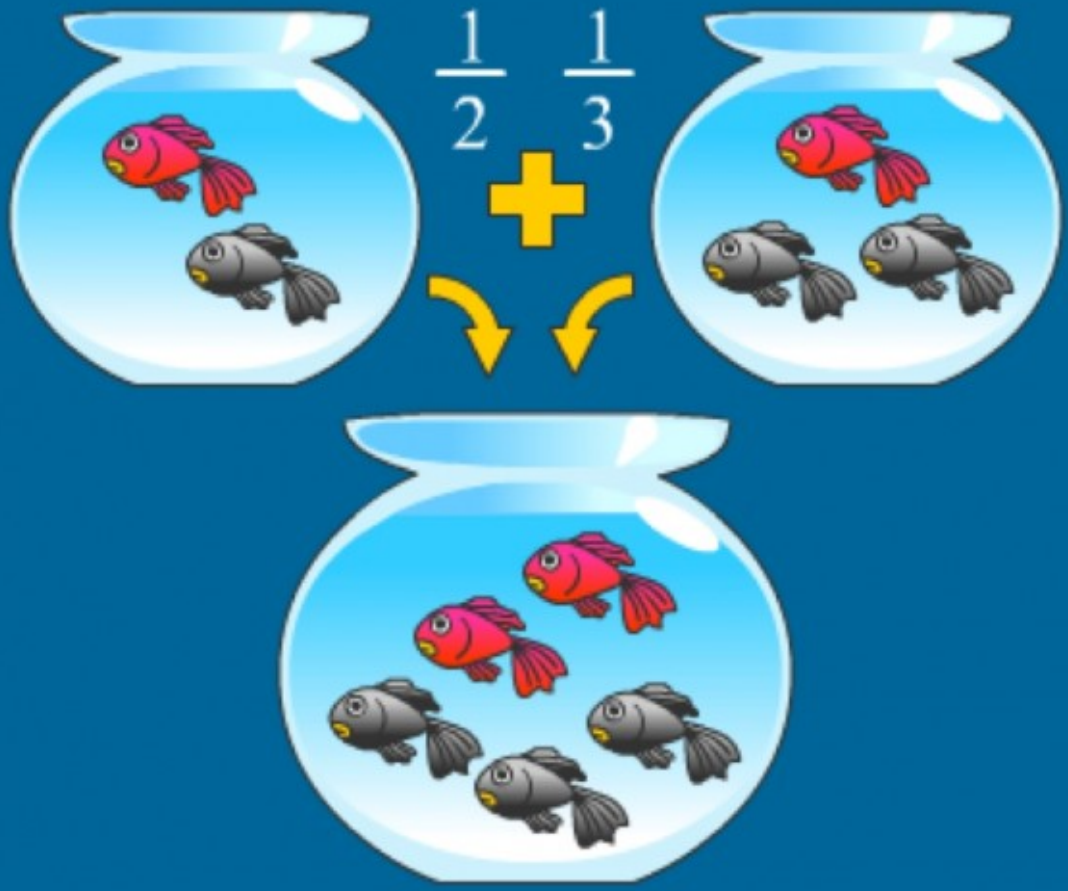
About how much?

A little more or a little less?

MATH SOLUTIONS ARTICLE?

"Student errors: What can they tell us about what students DO understand?"

赤い金魚は



赤い金魚は $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$?