Infinity Pizza

Geometry–Teacher Notes

Overview	
Students investigate slices of triangles to learn how midsegments and other divisions of a triangle affect the area.	**Prerequisite Understandings** • Find the area of a triangle both with the formula $A = \frac{b \cdot h}{2}$ and with Heron's formula. • Basic concepts of midsegments and formulas for distances between points.

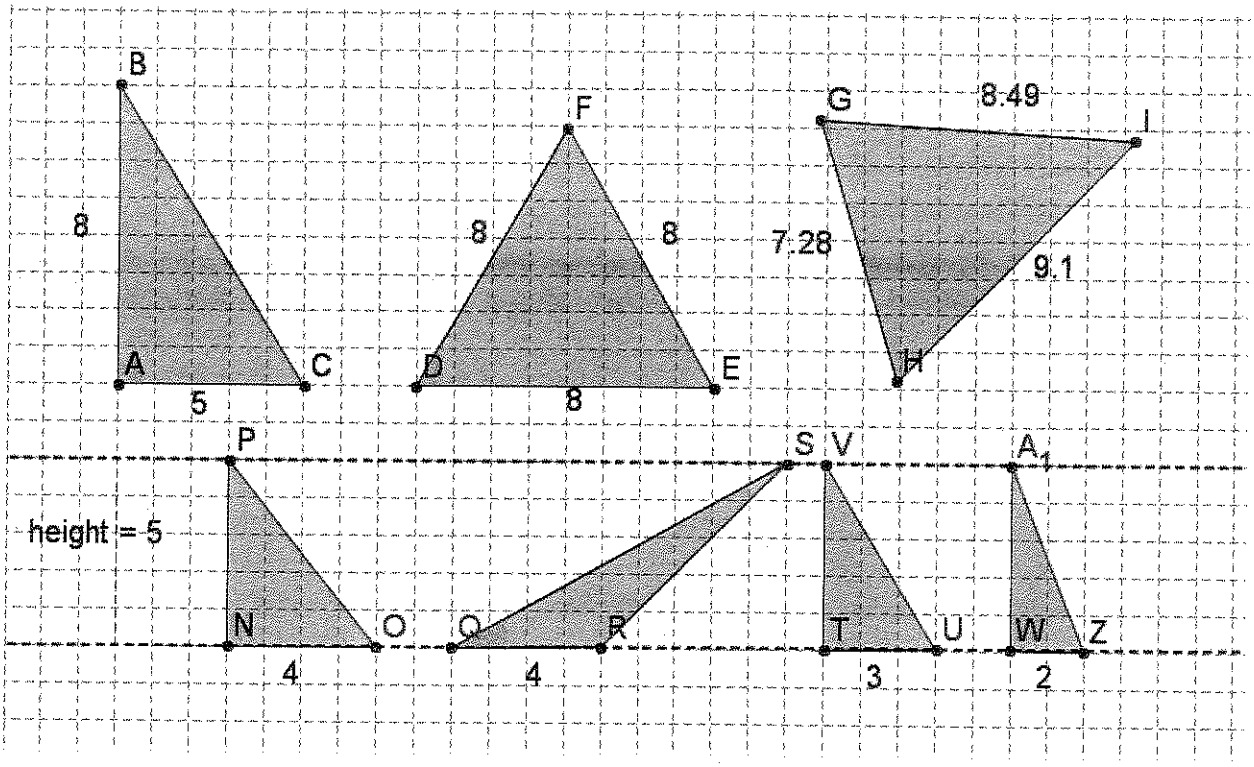
Curriculum Content	
CCSSM Content Standards	G.GPE.4. Use coordinates to prove simple geometric theorems algebraically. G.CO.10. Prove theorems about triangles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
CCSSM Mathematical Practices	2. **Reason abstractly and quantitatively**: Students reason based on formulas or on coordinates. 3. **Construct viable arguments and critique the reasoning of others**: Students explain and justify their reasoning in groups before sharing with the class. 8. **Look for and express regularity with repeated reasoning**: Students explore how changing inputs will change formulas.

Task	
Supplies • None	**Core Activity** Give students time to discuss, question, think, investigate, try, and revise their work. Set clear expectations for descriptions of thinking and logic.
Launch Students will think about triangular areas and their relationship to the length of sides, the height and base of various triangles.	**Extension(s)** Square, rectangular, trapezoidal, and other shapes of pizzas might be fun extensions. Students could explore, create, bake, and bring to school to eat!

Infinity Pizza

Launch

1. Find the areas of the following triangles using the traditional area formula, Heron's formula, and trigonometry.



2. How does changing the bases in the last problem to 3/4 or 1/2 of the original bases affect the areas?

3. If you cut the base in a triangle down to 1/3 of its original size while keeping the height the same, what will happen to the area?

4. If you cut the base in 1/2 and the height in 3/4, what will happen to the area of a triangle?

Infinity Pizza

Activity

Exploration 1: Sally's Pizza Chop Shop specializes in gigantic pizzas made in random triangle shapes. You and two good friends decide to get a pizza and share it equally.

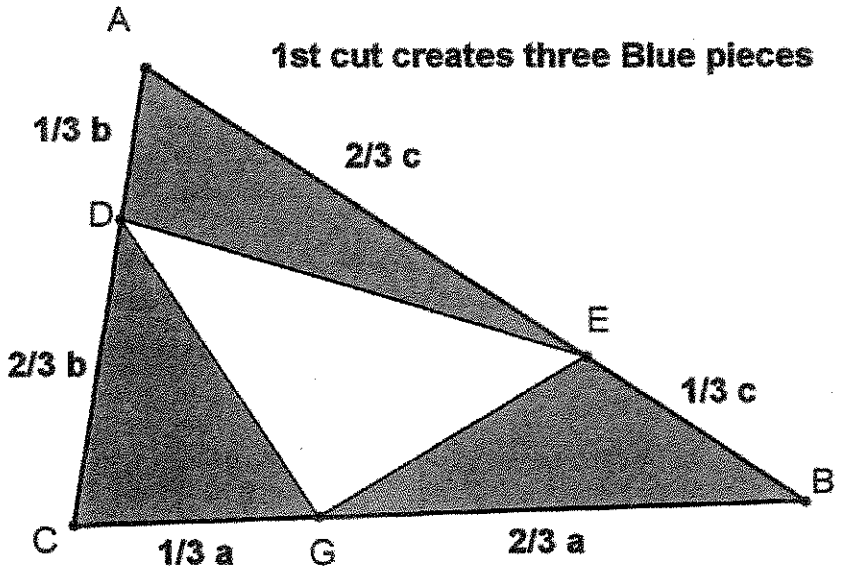
1. Devise a fair method for cutting any triangular pizza into 3 equal-sized pieces. Justify that your method creates a fair division using the formulas and properties of area for geometry.
2. Use an interactive geometry program or graphing calculator to model the problem and show that the areas are the same with that model.

Exploration 2: Sally uses her famous infinity cut to share a pizza with 3 people!

She first cuts along the lines from 1/3 of one side to 2/3 of the adjoining side. She claims that this gives each person an equal share ($1/3 \circ 2/3 = 2/9$) and leaves a triangular piece (1/3 of the pizza) for a second cut.

Then she cuts the remaining triangle in the same manner, leaving 1/3 of that triangle. The cutting goes on and on using that same pattern.

Sally exclaims, "Since the pieces get smaller and smaller, you get to eat less and less making it easier to finish her giant pizzas!" The cut method is illustrated below.



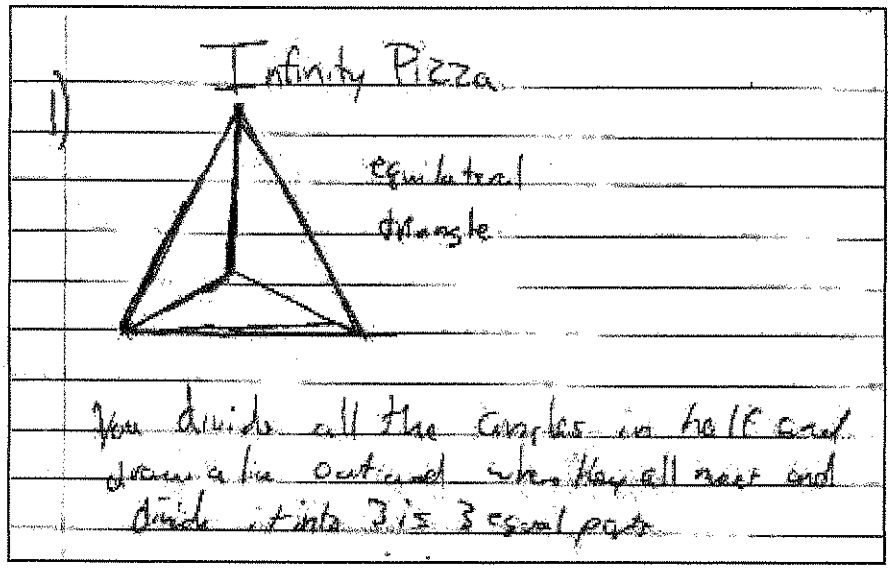
1. Explain why Sally's cut creates 3 equal shares.
2. Create a sequence of fractions that represent each person's share with each successive cut.
3. Does the sum of the sequence for each person's slices converge to 1/3? Explain.
4. Discuss the strengths and limitations to this method of pizza cutting and eating.

Infinity Pizza

Results from the Classroom

Exploration 1

The activity requires students to have a strong sense of how changing the base or height of a triangle affects the area. When given the first task of creating a strategy for cutting a triangular pizza into three equal shares, the most common response for a Geometry class just starting area is shown below.



An easy solution is to start with an equilateral triangle, bisect the angles, and (Voila!) problem solved.

Students found this nice simple solution of choosing an equilateral triangle also worked if they constructed the medians, or the altitudes, or the perpendicular bisectors. It appears that work on circumcenters, incenters, centroids, and orthocenters all had made some impression on the students.

The question "What about scalene triangles?" sent them back to the drawing board, and it appeared that they didn't have strong area strategies for dealing with this problem without some extensive work on how to divide segments into equal parts.

Some pre-activities involving dividing squares, rectangles, and then circles can help give them a sense on how changing a factor in a formula alters the final answer. Once they build a surer sense of how bases and altitudes determine the size of the area of a region, then they should have a better sense of how to solve and explain the problem.

Exploration 2

As to the infinity cuts, a Pre-Calculus student upon seeing the problem mentioned that it might be easier to use medians since that gives you four equal pieces. He said, "You can still share three pieces and go on sharing forever." It might be fun to see how much more pizza is left after each student consumes the first four diminishing pieces.

All of the students got a nice feel for limits as n approaches infinity. There is often that perceptive student who says just cut it from the vertex to a base divided in thirds (1/3) since the remaining piece is going to get real small real fast.

Quadrilaterals Flying High

Geometry—Teacher Notes

Overview	
Students investigate the properties of quadrilaterals.	**Prerequisite Understandings** • Basic vocabulary of quadrilaterals. • Congruency of triangles.

Curriculum Content	
CCSSM Content Standards	G.CO.10L. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
CCSSM Mathematical Practices	3. **Construct viable arguments and critique the reasoning of others:** Students make conjectures and justify their conclusions on an investigation of quadrilaterals. 4. **Model with mathematics:** Students will create scale drawings and use measures and transformations to justify properties of quadrilaterals and their diagonals.

Task	
Supplies • Graphing paper/Patty paper • Rulers (or tape measures) • Compasses • Tables (or large butcher block paper) for the original size kite	**Core Activity** Students explore kites by making a full-sized model (on tables using dry erase markers or butcher paper) and then making a scale model on with centimeter paper. Conjectures are made and checked using measurements and transformations.
Launch Students will make a scale drawing of a rectangle and be guided through the methods of creating and justifying conjectures.	**Extension(s)** This activity opens up room for exploration into making large models of many geometric shapes and discovering the properties of geometric figures.