Fractional Workers

Algebra—Teacher Notes

Overview	
Students solve fractional work problems with pictures, graphs, and equations of one or two variables.	**Prerequisite Understandings** • Graphing linear equations. • Writing equations in one or two variables. • Solving additive fractional problems.

Curriculum Content	
CCSSM Content Standards	A-CED.1. Create equations and inequalities in one variable and use them to solve problems. A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
CCSSM Mathematical Practices	4. Construct viable arguments and critique the reasoning of others: Students engage in rich discussions about the ways to interpret and model the concepts including using fractions and tables. 5. Model with mathematics: Students use pictorial, graphical, numerical, and algebraic models.

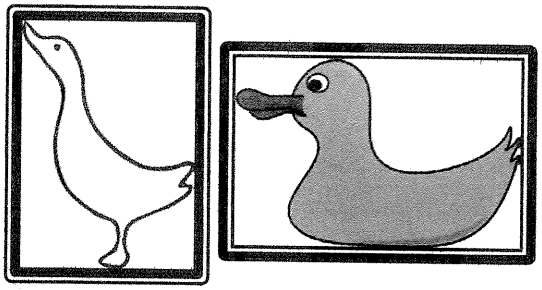
Task	
Supplies • Graphing paper • Graphing calculator	**Core Activity** Students explore the working rate of three students and investigate three models (pictorial, graphical, and algebraic) that describe working situations.
Launch Students practice reading and thinking about fractions on a graph and then graph work done vs. time.	**Extension(s)** Students could look at using 2 equations with 2 unknowns. After the activity, students should discuss, identify and solve rate problems in the form $\frac{t}{2}+\frac{t}{4}+\frac{t}{3.5}=1$ where the fractions represent the different speeds of the each worker.

Fractional Workers

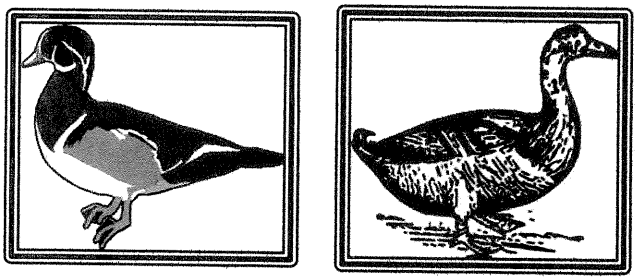
Launch

Otto and Sparky are famous "Duck" artists. They were bragging in front of Mary the Mathematician about how fast they can paint a duck picture and how much detail they included. The next day Mary presented them with a graph of time vs. paintings done. Examine the pictures, Mary's graphs, and answer the following questions. Otto painted the two pictures on the left, and Sparky painted the two on the right.

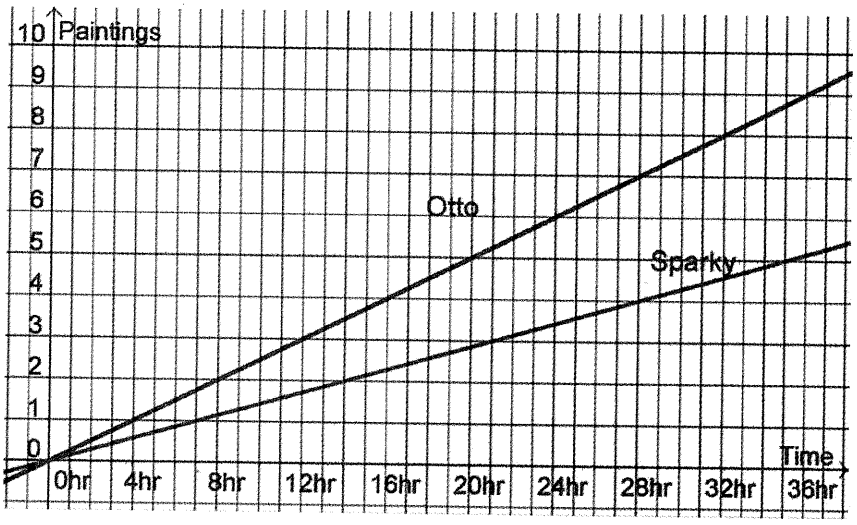
Otto



Sparky



Mary's Graph



1. Describe the graph.
2. Who paints fastest? How can you tell from the graph?
3. How fast is Otto?
4. How fast is Sparky?
5. Describe the axes.
6. Use the paintings to provide clues on why Sparky might be slower at finishing paintings.
7. Write a function based on time for both people. p=f(t)
8. If Sparky and Otto work together to paint 23 pictures for a benefit auction, how long will it take?
9. Explain how you got the answer so that the benefit auction organizers will understand.

Fractional Workers

Activity

Sally, Sarah, and Suzie run the **S³ Painting Service** where they paint old one-car garages for a summer job. They have lots of work because they live in an old part of town with lots of garages, and their fees are pretty cheap. They paint garages for less because they just love to fix up old garages so they look beautiful and elegant and make the world a better place. They also are saving lots of money for college!

Over the past few summers, they have figured out that speedy Sally can paint a garage in 2 days. Slower Sarah takes twice as long as Sally. Somehow Suzie can paint a garage by herself half a day faster than Sarah can paint one by herself.

Help us analyze this business.

1. How fast can each person paint a garage by themselves?

2. Make a sketch of 20 garages and show how much each girl will have painted after 7 days and how long will it take them to finish them all. Explain how you got your answer.

3. Create equations and make linear graphs that show how speedy Sally, slower Sarah, and somehow Suzie compare in their painting speeds. Let y represent the number of garages and x represent the time in days.

4. Figure how long would it take for Sally and Sarah to paint one garage together. Explain how you got your answer.

5. The **S³** girls have decided they can make a profit of $280 on each garage and that they will share the profits equally. Show how to figure out how much money they can make in a regular summer.

Fractional Workers

Results from the Classroom

Day	Sally	Sarah	Suzie
1			
2	✓		
3			
3 ½			✓
4	✓	✓	
5			
6	✓		
7			✓
8	✓	✓	
9			
10	✓		
10 ½			✓
11			
12	✓	✓	
13			
14	✓		✓

One of the students created a table of days with tally marks like the one at left. Hayley kept a tally of garages that were painted and just kept going until she got the correct number of garages. The table on the left illustrates the method, and you can see that by the 14th day that there were 14 garages painted. She was the first one to confidently get an exact day for her answer rather than a fractional day answer that she could justify.

Most students were fairly proficient is giving straight forward answers like the one below right. The one mistake made here came right at the beginning with assuming Suzie would take 4 ½ days vs. the actual 3 ½ from the description. This solution demonstrates how the first approach most students take is arithmetic.

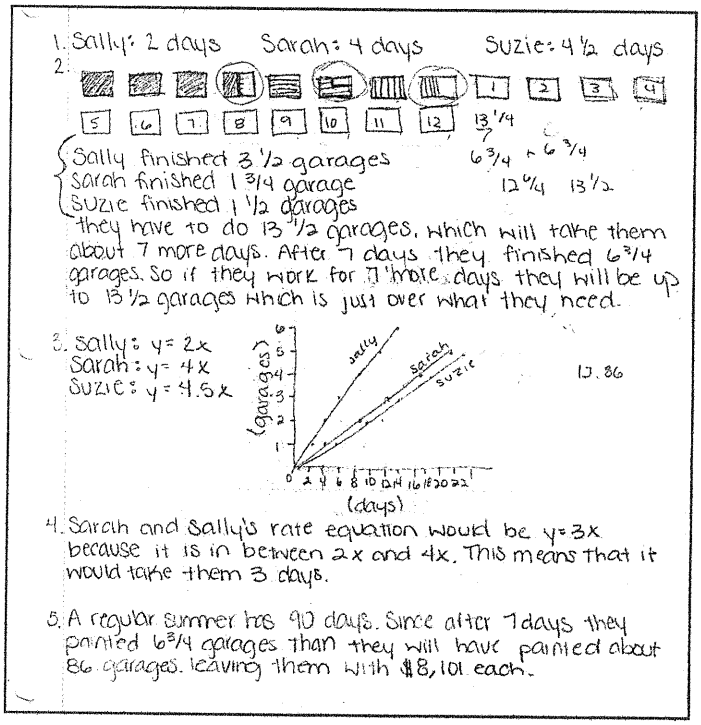
It was expected that most students would use an algebraic solution like the one below, but it was rare to have it approached in the traditional manner.

$$\frac{t}{4}+\frac{t}{3.5}+\frac{t}{2}=20$$

$$\frac{7t}{28}+\frac{8t}{28}+\frac{14t}{28}=20$$

$$\frac{29t}{28}=20$$

$$\therefore t \approx 19.3 \; days$$



The students used a variety of approaches and could engage in the practices by sharing their different approaches. If none of the students use the algebraic solution in your classroom, it could an opportunity to share it with them and encourage them to compare it to the other approaches generated.